EXERCISE – III HINTS & SOLUTIONS

Sol.1 (i) $I = \int \sin 2x\, dx + \int \frac{dx}{x+1}$

$= -\frac{\cos 2x}{2} + \ln(x+1) + c$

(ii) $I = \int \tan(3x+1)\, dx + \int e^{4x+5}\, dx$

$= \frac{1}{3}\ \ell n \sec(3x+1) + \frac{1}{4} e^{4x+5} + c$

(iii) $I = 2\int \tan(4x+5)\, dx$

$= \frac{2}{4}\ \ell n \sec(4x+5) + c$

$= \frac{1}{2}\ \ell n \sec(4x+5) + c$

(iv) $I = \int \frac{x+2-2}{\sqrt{x+2}}\, dx$

$= \int \sqrt{x+2}\, dx - 2\int (x+2)^{-1/2}\, dx$

$= \frac{2}{3}(x+2)^{3/2} - 4\sqrt{x+2} + c$

(v) $I = \int \sin^2 x\, dx$

$= \int \left(\frac{1-\cos 2x}{2}\right) dx$

$= \frac{1}{2}x - \frac{\sin 2x}{4} + c$

(vi) $I = \int \cos^2 x\, dx$

$= \int \left(\frac{1+\cos 2x}{2}\right) dx$

$= \frac{1}{2}x + \frac{\sin 2x}{4} + c$

(vii) $I = \frac{1}{2}\int 2\sin 2x \cos 3x\, dx$

$= \frac{1}{2}\int (\sin 5x - \sin x)\, dx$

$= \frac{1}{2}\left[-\frac{\cos 5x}{5} + \cos x\right] + c$

Aliter

$I = 2\int \sin x.\cos x.\cos 3x\, dx$

$= 2\int \sin x.\cos x(4\cos^3 x - 3\cos x)\, dx$

put $\cos x = t$

(viii) $I = \int \left(e^x + \frac{1}{e^x}\right)^2 dx$

$= \int \left(e^{2x} + e^{-2x} + 2\right) dx$

$= \frac{e^{2x}}{2} - \frac{e^{-2x}}{2} + 2x + c$

(ix) $I = \int (e^x + 1)^2 e^x\, dx$

$= \int (e^{2x} + 1 + 2e^x)e^x\, dx$

$= \int (e^{3x} + e^x + 2e^{2x})\, dx$

$= \frac{e^{3x}}{3} + e^x + e^{2x} + c$

(x) $I = \int \frac{\sqrt{x+3} + \sqrt{x+2}}{(x+3)-(x+2)}\, dx$

$= \int (\sqrt{x+3} + \sqrt{x+2})\, dx$

$= \frac{2}{3}[(x+3)^{3/2} + (x+2)^{3/2}] + c$

Sol.2 (i) $I = \int x \sin x^2\, dx$

Let $x^2 = t \Rightarrow xdx = \frac{dt}{2}$

$= \frac{1}{2}\int \sin t\, dt = -\frac{1}{2}\cos x^2 + c$

(ii) $I = \frac{1}{2}\int \frac{2x}{x^2+1}\, dx$

$= \frac{1}{2}\ \ell n (x^2+1) + c$

(iii) $I = \int \sec^2 x \tan x\, dx$

$= \frac{\tan^2 x}{2} + c$

(iv) $I = \int \frac{e^x + 1}{e^x + x} dx = \ell n (e^x + x) + c$

(v) $I = \int \frac{1 - \sin x}{x + \cos x} dx$

Let $x + \cos x = t \Rightarrow (1 - \sin x)\, dx = dt$

$= \int \frac{dt}{t} = \ell n (x + \cos x) + c$

(vi) $I = \int \frac{e^{2x}}{e^{2x} - 2} dx$

Let $e^{2x} - 2 = t \Rightarrow e^{2x} dx = \frac{dt}{2}$

$= \frac{1}{2} \int \frac{dt}{t} = \frac{1}{2} \ell n (e^{2x} - 2) + c$

(vii) $I = \int \frac{\cos 2x + x + 1}{x^2 + \sin 2x + 2x} dx$

Let $x^2 + \sin 2x + 2x = t$

$(2x + 2 \cos 2x + 2)\, dx = dt$

$(\cos 2x + x + 1)\, dx = \frac{dt}{2}$

$= \frac{1}{2} \int \frac{dt}{t} = \frac{1}{2} \ell n (x^2 + \sin 2x + 2x) + c$

(viii) $I = \int \frac{\sec x}{\ell n (\sec x + \tan x)} dx$

Let $\ell n (\sec x + \tan x) = t$

$\frac{\sec x(\tan x + \sec x)}{(\tan x + \sec x)} dx = dt$

$\sec x\, dx = dt$

$= \int \frac{dt}{t} = \ell n\, \ell n (\sec x + \tan x) + c$

(ix) $I = \int x^5 \sqrt{a^3 + x^3}\, dx$

$= \int x^3 . x^2 \sqrt{a^3 + x^3}\, dx$

Let $a^3 + x^3 = t^2 \Rightarrow x^2 dx = \frac{2t\, dt}{3}$

$= \int (t^2 - a^3)\, t . \left(\frac{2t}{3}\right) dt$

$= \frac{2}{3} \int (t^4 - a^3 t^2)\, dt = \frac{2}{3} \left[\frac{t^5}{5} - \frac{a^3\, t^3}{3}\right] + c$

where $t = \sqrt{a^3 + x^3}$

Sol.3 **(i)** $I = \int x \sin x\, dx$

$= - x \cos x + \int \cos x\, dx$

$= \sin x - x \sin x + c$

(ii) $I = \int x . \ell n\, x\, dx$

$= \frac{x^2}{2}\, \ell n\, x - \int \frac{x^2}{2} . \frac{1}{x} dx$

$= \frac{x^2}{2}\, \ell n\, x - \frac{x^2}{4} + c$

(iii) $I = \int x \sin^2 x\, dx$

$= \int x \left(\frac{1 - \cos 2x}{2}\right) dx$

$= \frac{x^2}{4} - \frac{1}{2} \int x \cos 2x\, dx$

$= \frac{x^2}{4} - \frac{1}{2} \left[\frac{x \sin 2x}{2} - \frac{1}{2} \int \sin 2x\, dx\right]$

$= \frac{x^2}{4} - \frac{1}{4} x \sin 2x - \frac{1}{8} \cos 2x + c$

(iv) $I = \int x . \tan^{-1} x\, dx$

$= \frac{x^2}{2} \tan^{-1} x - \frac{1}{2} \int \frac{x^2}{1 + x^2} dx$

$= \frac{x^2}{2} \tan^{-1} x - \frac{1}{2} x + \frac{1}{2} \tan^{-1} x + c$

(v) $I = \int 1 . \ell n\, x\, dx = x . \ell n\, x - x + c$

(vi) $I = \int \sec^3 x\, dx = \int \sec x . \sec^2 x\, dx$

$= \sec x . \tan x - \int \sec x\, (\sec^2 x - 1)\, dx$

$I = \sec x . \tan x - I + \int \sec x\, dx$

$2I = \sec x . \tan x + \ell n (\sec x + \tan x) + c$

$I = \frac{1}{2} [\sec x \tan x + \ell n (\sec x + \tan x)] + c$

(vii) $I = \int 2x^2 . x\, e^{x^2} dx$

Let $x^2 = t \Rightarrow 2x\, dx = dt$

$= \int t . e^t\, dt = t . e^t - e^t + c$

$= x^2 e^{x^2} - e^{x^2} + c$

(viii) $I = \int 1.\sin^{-1}\sqrt{x}\, dt$

$= x\,.\sin^{-1}\sqrt{x} - \int \frac{x}{\sqrt{1-x}} dx.\frac{1}{2\sqrt{x}}$

$= x\,.\sin^{-1}\sqrt{x} - \frac{1}{2}\int \frac{\sqrt{x}}{\sqrt{1-x}} dx$

$= x\,.\sin^{-1}\sqrt{x} + \frac{\sqrt{x}\sqrt{1-x}}{2} - \frac{\sin^{-1}\sqrt{x}}{2} + c$

(ix) $I = \int \frac{x^2 \tan^{-1} x}{1+x^2} dx$

Put $\tan^{-1} x = t \Rightarrow \frac{dx}{1+x^2} = dt$

$= \int t\,.\tan^2 t\, dt = \int t\,(\sec^2 t - 1)\, dt$

$= \int \sec^2 t - \int \tan t - \frac{t^2}{2} + c$

$= t \tan t - \ell n\,(\sec t) - \frac{t^2}{2} + c$

$= x \tan^{-1}x - \ell n\,(\sec(\tan^{-1} x)) - \frac{(\tan^{-1}x)^2}{2} + c$

(x) $\int e^x \sin x\, dx = \frac{e^x}{2}(\sin x - \cos x) + c$

(xi) $\int (e^x(\sec^2 x + \tan x)\, dx$

$= e^x \tan x - \int e^x \tan x\, dx + \int e^x \tan x\, dx$

$= e^x \tan x + c$

Sol.4 **(i)** $I = \int \sqrt{x^2+4}\, dx$

$= \frac{x}{2}\sqrt{x^2+4} + \frac{4}{2}\ell n\,(x + \sqrt{x^2+4}) + c$

(ii) $I = \int \frac{dx}{x^2+4} = \frac{1}{2}\tan^{-1}\frac{x}{2} + c$

(iii) $I = \int \frac{dx}{\sqrt{x^2-4}} = \ell n\,(x + \sqrt{x^2-4}) + c$

(iv) $I = \int \frac{dx}{x^2+5} = \frac{1}{\sqrt{5}}\tan^{-1}\frac{x}{\sqrt{5}} + c$

(v) $I = \int \sqrt{x^2+2x+5}\, dx$

$= \int \sqrt{(x+1)^2+(2)^2}\, dx$

$= \frac{x+1}{2}\sqrt{x^2+2x+5} + 2\ell n\,|\,(x+1)+\sqrt{x^2+2x+5}\,| + c$

(vi) $I = \int \frac{dx}{(x+1)^2+4} = \frac{1}{2}\tan^{-1}\left(\frac{x+1}{2}\right) + c$

(vii) $I = \int (x-1)\sqrt{1-x-x^2}\, dx$

put $x - 1 = A\frac{d}{dx}(1 - x - x^2) + B$

$x - 1 = A(-1 - 2x) + B$

$1 = -2A \Rightarrow A = -\frac{1}{2}$

$-1 = -A + B \Rightarrow B = -\frac{3}{2}$

$I = A\int (-1-2x)\sqrt{1-x-x^2}\, dx + B\int \sqrt{1-x-x^2}\, dx$

(viii) $I = \int \frac{2x+1}{x^2+3x+4} dx$

$2x + 1 = A(2x + 3) + B$

$2 = 2A \Rightarrow A = 1$

$1 = 3A + B \Rightarrow B = -2$

$I = \int \frac{2x+3}{x^2+3x+4} - 2\int \frac{dx}{x^2+3x+4}$

$= \ell n\,(x^2 + 3x + 4) - \frac{4}{\sqrt{7}}\tan^{-1}\frac{2x+3}{\sqrt{7}} + c$

(ix) $I = \int \frac{dx}{x(x^5+1)} = \int \frac{dx}{x^6(1+x^{-5})}$

$1 + x^{-5} = t \Rightarrow \frac{dx}{x^6} = -\frac{dt}{5}$

$= -\frac{1}{5}\int \frac{dt}{t} = -\frac{1}{5}\ell n\, t = -\frac{1}{5}\ell n\,(1 + x^{-5}) + c$

(x) $I = \int \frac{1}{x^5(1+x^5)^{1/5}} = \int \frac{1}{x^6(1+x^{-5})^{1/5}}$

$1 + x^{-5} = t^5 \Rightarrow \frac{dx}{x^6} = -t^4\, dt$

$= \int \frac{-t^4\, dt}{t} = -\int t^3\, dt = -\frac{t^4}{4} + c$

$= -\frac{1}{4}(1 + x^{-5})^{4/5} + c$

(xi) $I = \int \frac{\sqrt{x^2-8}}{x^4}\,dx = \int \frac{1}{x^3}\sqrt{1-8x^{-2}}\;dx$

$1 - 8x^{-2} = t^2 \quad \Rightarrow \quad \frac{dx}{x^3} = \frac{t}{8}\,dt$

$\frac{1}{8}\int t\,t\,dt = \frac{t^3}{24} + c = \frac{1}{24}\,(1 - 8x^{-2})^{3/2} + c$

(xii) $I = \int \frac{x^3-1}{x(x^2+1)}\,dx$

$= \int \frac{x^2+1-1}{x^2+1}\,dx - \int \frac{dx}{x^3(1+x^{-2})}$

$= \int dx - \int \frac{dx}{1+x^2} - \int \frac{dx}{x^3(1+x^{-2})}$

$= x - \tan^{-1} x$; Let $1 + x^{-2} = t$

$\frac{dx}{x^3} = \frac{-1}{2}\,dt$

$= x - \tan^{-1} x + \frac{1}{2}\int \frac{dt}{t}$

$= x - \tan^{-1} x + \frac{1}{2}\,\ell n\,(1 + x^{-2}) + c$

Sol.5 **(i)** $I = \int \frac{dx}{2+\cos x}$

$= \int \frac{\sec^2\frac{x}{2}\,dx}{2\left(1+\tan^2\frac{x}{2}\right)+1-\tan^2\frac{x}{2}} = \int \frac{\sin^2\frac{x}{2}\,dx}{3+\tan^2\frac{x}{2}}$

Let $\tan\frac{x}{2} = t \Rightarrow \sec^2\frac{x}{2}\,dx = 2dt$

$= \int \frac{2\,dt}{3+t^2} = \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{\tan\frac{x}{2}}{\sqrt{3}}\right) + c$

(ii) $I = \int \frac{dx}{2-\cos x}$

$= \int \frac{\sec^2\frac{x}{2}\,dx}{2\left(1+\tan^2\frac{x}{2}\right)-\left(1-\tan^2\frac{x}{2}\right)}\,dx$

Let $\tan\frac{x}{2} = t \Rightarrow \sec^2\frac{x}{2}\,dx = 2dt$

$= \int \frac{2\,dt}{2(1+t^2)-(1-t^2)} = 2\int \frac{dt}{1+3t^2}$

$= \frac{2}{3}\int \frac{dt}{t^2+\left(\frac{1}{\sqrt{3}}\right)^2}$

$= \frac{2}{3\left(\frac{1}{\sqrt{3}}\right)}\tan^{-1}\left(\sqrt{3}\tan\frac{x}{2}\right) + c$

$= \frac{2}{\sqrt{3}}\tan^{-1}\left(\sqrt{3}\tan\frac{x}{2}\right) + c$

(iii) $I = \int \frac{2\sin x + 2\cos x}{3\cos x + 2\sin x}\,dx$

Let $2\sin x + 2\cos x = A(D^r) + B\frac{d}{dx}(D^r)$

$2\sin x + 2\cos x = A(3\cos x + 2\sin x) + B(2\cos x - 3\sin x)$

$2 = 2A - 3B$

$2 = 3A + 2B$

$A = \frac{10}{13}$; $B = -\frac{2}{13}$

$I = \frac{10}{13}x - \frac{2}{13}\,\ell n\,(3\cos x + 2\sin x) + c$

(iv) $I = \int \frac{dx}{1+\sin x+\cos x}$

$= \int \frac{\sec^2\frac{x}{2}\,dx}{1+\tan^2\frac{x}{2}+2\tan\frac{x}{2}+1-\tan^2\frac{x}{2}}$

Let $\tan\frac{x}{2} = t \Rightarrow \sec^2\frac{x}{2}\,dx = 2dt$

$= \int \frac{2\,dt}{1+2t} = \ell n\left(1+2\tan\frac{x}{2}\right) + c$

(v) $I = \int \frac{dx}{2+\sin^2 x} = \int \frac{dx}{2+\frac{1-\cos 2x}{2}}$

$= \int \frac{2\,dx}{5-\cos 2x}$

Next is similar form as (i) we can solve

(vi) $\int \frac{\operatorname{cosec}^2 x \sin x}{(\sin x - \cos x)} dx = \int \frac{1}{\sin x(\sin x - \cos x)} dx$

$= \int \frac{\sec^2 x}{\tan x(\tan x - 1)} dx$

put $\tan x = t \Rightarrow \sec^2 x\, dx = dt$

$= \int \frac{dt}{t(t-1)} = \int \frac{t-(t-1)}{t(t-1)} dt$

$= \int \frac{dt}{t-1} - \int \frac{dt}{t} = \ell n \left(\frac{t-1}{t}\right) + c$

$= \ell n \left(\frac{\tan x - 1}{\tan x}\right) + c = \ell n \left(\frac{\sin x - \cos x}{\sin x}\right) + c$

(vii) $I = \int \frac{\sin^4 x}{\cos^2 x} dx = \int \tan^2 x \cdot \sin^2 x\, dx$

$= \int (\sec^2 x - 1) \sin^2 x\, dx$

$= \int \tan^2 x\, dx - \int \sin^2 x\, dx$

$= \int (\sec^2 x - 1) dx - \int \left(\frac{1-\cos 2x}{2}\right) dx$

$= \tan x - x - \frac{x}{2} + \frac{\sin 2x}{4} + c$

$= \tan x - \frac{3x}{2} + \frac{\sin 2x}{4} + c$

Sol.6 **(i)** $I = \int \frac{dx}{x^4 + x^2 + 1} = \frac{1}{2} \int \frac{x^2+1+1-x^2}{x^4+x^2+1} dx$

$= \frac{1}{2} \int \frac{x^2+1}{x^4+x^2+1} dx - \frac{1}{2} \int \frac{x^2-1}{x^4+x^2+1} dx$

$= \frac{1}{2} \int \frac{1+\frac{1}{x^2}}{x^2+1+\frac{1}{x^2}} dx - \frac{1}{2} \int \frac{1-\frac{1}{x^2}}{x^2+1+\frac{1}{x^2}} dx$

$= \frac{1}{2} \int \frac{1+\frac{1}{x^2}}{\left(x-\frac{1}{x}\right)^2+3} dx - \frac{1}{2} \int \frac{1-\frac{1}{x^2}}{\left(x+\frac{1}{x}\right)^2} dx$

$= \frac{1}{2} \int \frac{dt}{t^2+3} - \frac{1}{2} \int \frac{dt}{t^2-1}$

$= \frac{1}{2\sqrt{3}} \tan^{-1} \frac{t}{\sqrt{3}} - \frac{1}{4} \ell n \frac{t-1}{t+1} + c$

$= \frac{1}{2\sqrt{3}} \tan^{-1} \frac{1}{\sqrt{3}} \left(x - \frac{1}{x}\right) - \frac{1}{4} \ell n \frac{x+\frac{1}{x}-1}{x+\frac{1}{x}+1} + c$

(ii) $I = \int \frac{1+x^2}{1+x^4} dx = \int \frac{1+\frac{1}{x^2}}{x^2+\frac{1}{x^2}} dx$

$= \int \frac{1+\frac{1}{x^2}}{\left(x-\frac{1}{x}\right)^2+2} dx$

Let $x - \frac{1}{x} = t \Rightarrow \left(1+\frac{1}{x^2}\right) dx = dt$

$= \int \frac{dt}{t^2+2} = \frac{1}{\sqrt{2}} \tan^{-1} \frac{t}{\sqrt{2}} + c$

(iii) $I = \int \frac{1-x^2}{x^4-x^2+1} dx$

$= \int \frac{\frac{1}{x^2}-1}{x^2-1+\frac{1}{x^2}} dx = -\int \frac{1-\frac{1}{x^2}}{\left(x+\frac{1}{x}\right)^2-3} dx$

$= -\frac{1}{2\sqrt{3}} \ell n \frac{x+\frac{1}{x}-\sqrt{3}}{x+\frac{1}{x}+\sqrt{3}} + c$

Sol.7 **(i)** $I = \int \frac{1}{(x+1)\sqrt{x+2}} dx$

Let $x + 2 = t^2 \Rightarrow dx = 2t\, dt$

$= \int \frac{2t\, dt}{(t^2-1)\, t} = 2\int \frac{dt}{(t^2-1)}$

$= 2\int \frac{dt}{(t+1)(t-1)} = \int \frac{(t+1)-(t-1)}{(t+1)(t-1)} dt$

$= \int \frac{dt}{t-1} - \int \frac{dt}{t+1} = \ell n \frac{t-1}{t+1} + c$

$= \ell n \left| \frac{\sqrt{x+2}-1}{\sqrt{x+2}+1} \right| + c$

(ii) $I = \int \frac{1}{(x^2-4)\sqrt{x+1}}\,dx$

put $x + 1 = t^2 \Rightarrow dx = 2t\,dt$

$= 2\int \frac{2t\,dt}{\left[(t^2-1)^2-4\right]t}$

$= 2\int \frac{dt}{(t^2-1+2)(t^2-1-2)}$

$= 2\int \frac{dt}{(t^2+1)(t^2-3)} = \frac{2}{4}\int \frac{(t^2+1)-(t^2-3)}{(t^2+1)(t^2-3)}dt$

$= \frac{1}{2}\int \frac{dt}{t^2-3} - \frac{1}{2}\int \frac{dt}{t^2+1}$

$= \frac{1}{4\sqrt{3}}\ \ell n\ \frac{t-\sqrt{3}}{t+\sqrt{3}} - \frac{1}{2}\tan^{-1} t + c$

(iii) $I = \int \frac{dx}{(x+1)\sqrt{x^2+2}}$

put $x + 1 = \frac{1}{t} = \Rightarrow dx = \frac{-1}{t^2}dt$

$\Rightarrow \int \frac{(-1)dt}{t^2\left(\frac{1}{t}\right)\sqrt{\left(\frac{1}{t}-1\right)^2+2}} \Rightarrow \int \frac{-dt}{\sqrt{1+3t^2-2t}}$

$\Rightarrow \frac{-1}{\sqrt{3}}\int \frac{dt}{\sqrt{t^2-\frac{2}{3}t+\frac{1}{3}}}$

$\Rightarrow -\frac{1}{\sqrt{3}}\int \frac{dt}{\sqrt{t^2-\frac{2}{3}t+\frac{1}{9}-\frac{1}{9}+\frac{1}{3}}}$

$\Rightarrow -\frac{1}{\sqrt{3}}\int \frac{dt}{\sqrt{\left(t-\frac{1}{3}\right)^2+\frac{2}{9}}}$

$\Rightarrow -\frac{1}{\sqrt{3}}\ \ell n \left|\left(t-\frac{1}{3}\right)+\sqrt{\left(t-\frac{1}{3}\right)^2+\frac{2}{9}}\right| + c$

where, $t = \frac{1}{x+1}$

(iv) $I = \int \frac{dx}{(x^2+1)\sqrt{x^2+2}}$

put $x = 1/t \Rightarrow dx = -\frac{dt}{t^2}$

$= \int \frac{1}{\left(\frac{1}{t^2}+1\right)\sqrt{\frac{1}{t^2}+2}}\cdot\frac{-dt}{t^2}$

$= -\int \frac{t}{(t^2+1)\sqrt{2t^2+1}}dt$

$2t^2 + 1 = z^2 \Rightarrow tdt = \frac{z}{2}dz$

$= -\frac{1}{2}\int \frac{z\,dz}{\left(\frac{z^2-1}{2}+1\right)z} = -\int \frac{dz}{(z^2+1)}$

$= -\tan^{-1}\sqrt{2t^2+1} + c$

Sol.8 (i) $I = \int \frac{dx}{(x+1)(x+2)}$

$= \int \frac{(x+2)-(x+1)}{(x+1)(x+2)}dx$

$= \int \frac{dx}{x+1} - \int \frac{dx}{x+2} = \ell n\ \frac{x+1}{x+2} + c$

(ii) $I = \int \frac{dx}{(x^2+1)(x+3)}$

$\frac{1}{(x^2+1)(x+3)} = \frac{A}{(x+3)} + \frac{Bx+C}{x^2+1}$

$1 = A(x^2 + 1) + (Bx + C)(x + 3)$

put $x = -3$; $1 = 10A \Rightarrow A = \frac{1}{10}$

put $x = 0$; $1 = A + 3C \Rightarrow 3C = 1 - \frac{1}{10}$

$3C = \frac{9}{10} \Rightarrow C = \frac{3}{10}$

put $x = 1$; $1 = 2A + 4(B + C)$

$B = -\frac{1}{10}$

$$I = \frac{1}{10}\int \frac{dx}{x+3} - \frac{1}{10}\int \frac{x\,dx}{x^2+1} + \frac{3}{10}\int \frac{dx}{x^2+1}$$

$$= \frac{1}{10}\ell n\,(x+3) - \frac{1}{20}\ell n\,(x^2+1) + \frac{3}{10}\tan^{-1}x + c$$

(iii) $I = \int \frac{dx}{(x+1)^2(x+2)}$

$$\frac{1}{(x+1)^2(x+2)} = \frac{A}{x+2} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$$

$1 = A(x + 1)^2 + B(x + 2)(x + 1) + C(x + 2)$

put $x = -1 \Rightarrow c = 1$

put $x = -2 \Rightarrow A = 1$

$x = 0 \quad 1 = A + 2B + 2C$

$1 = 1 + 2B + 2 \Rightarrow B = -1$

$$I = \int \frac{dx}{x+2} - \int \frac{dx}{x+1} + \int \frac{dx}{(x+1)^2}$$

$$= \ell n \left(\frac{x+2}{x+1}\right) - \frac{1}{(x+1)} + c$$

(iv) $I = \int \frac{dx}{(x+1)(x+2)(x+3)}$

$$\frac{1}{(x+1)(x+2)(x+3)} = \frac{A}{x+1} + \frac{B}{x+2} + \frac{C}{x+3}$$

$1 = A(x + 2)(x + 3) + B(x + 1)(x + 3) + C(x + 1)(x + 2)$

put $x = -1$; $1 = 2A \Rightarrow A = 1/2$

put $x = -2$; $1 = -B \Rightarrow B = -1$

put $x = -3$; $1 = C(-2)(-1) \Rightarrow C = \frac{1}{2}$

$$I = \frac{1}{2}\ell n\,(x+1) - \ell n(x+2) + \frac{1}{2}\ell n(x+3) + c$$

Sol.9 $I = \int \sin^2 x \cos^2 x\,dx$

$$= \frac{1}{4}\int \sin^2 2x\,dx = \frac{1}{4}\int \left(\frac{1-\cos 4x}{2}\right)dx$$

$$= \frac{1}{8}x - \frac{1}{32}\sin 4x + c$$

Sol.10 $I = \int \frac{dx}{\sin(x-a)\cos(x-b)}$

$$I = \frac{1}{\cos(b-a)}\int \frac{\{(x-a)-(x-b)\}}{\sin(x-a)\cos(x-b)}dx$$

$$= \frac{1}{\cos(b-a)}\int \frac{\cos(x-a)\cos(x-b)+\sin(x-a)\sin(x-b)}{\sin(x-a)\cos(x-b)}dx$$

$$= \frac{1}{\cos(b-a)}\int \left[\cot(x-a)+\tan(x-b)\right]dx$$

$$= \frac{1}{\cos(b-a)}\left[\ell n \sin(x-a) + \ell n \sec(x-b)\right] + c$$

$$= \frac{1}{\cos(b-a)}\left[\ell n\left|\frac{\sin(x-a)}{\cos(x-b)}\right|\right] + c$$

Sol.11 $\int \frac{x+\sqrt{x+1}}{x+2}dx$

put $x + 1 = t^2 \Rightarrow dx = 2t\,dt$

$$= \int \left[\frac{t^2-1+t}{t^2+1}\right]2t\,dt = \int \left(\frac{t^2+1+t-2}{t^2+1}\right)2t\,dt$$

$$= 2\int t\,dt + 2\int \frac{t^2+1}{t^2+1}dt - 2\int \frac{dt}{t^2+1} - 2\int \frac{2t}{t^2+1}dt$$

$= t^2 + 2t - 2\tan^{-1} t - 2\,\ell n\,(1 + t^2) + c$

Sol.12 $I = \int \frac{x^2+1-2x}{(x^4+x^2+1)}dx$

$$= \int \frac{x^2+1}{x^4+x^2+1}dx - \int \frac{2x\,dx}{x^4+x^2+1}$$

put $x^2 = t \Rightarrow 2x\,dx = dt$

$$= \int \frac{1+\frac{1}{x^2}}{x^2+1+\frac{1}{x^2}}dx - \int \frac{dt}{t^2+t+1+\frac{1}{4}-\frac{1}{4}}$$

$$= \int \frac{1+\frac{1}{x^2}}{\left(x-\frac{1}{x}\right)^2+3}dx - \int \frac{dt}{\left(t+\frac{1}{2}\right)^2+\frac{3}{4}}$$

$$= \int \frac{dz}{z^2+3} - \int \frac{dt}{\left(t+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}$$

$$= \frac{1}{\sqrt{3}}\tan^{-1}\left(\left(x-\frac{1}{x}\right)\frac{1}{\sqrt{3}}\right) - \frac{1}{\frac{\sqrt{3}}{2}}\tan^{-1}\left[\left(x^2+\frac{1}{2}\right)\frac{2}{\sqrt{3}}\right] + c$$

Sol.13 $I = \int \frac{x\,\ell n\,x}{(x^2-1)^{3/2}}\,dx$

$= \int \underbrace{\frac{x}{(x^2-1)^{3/2}}}_{II}\,\underbrace{\ell n\,x}_{I}\,dx$

$= \ell n\,x \int \frac{x\,dx}{(x^2-1)^{3/2}} - \int \frac{1}{x} \int \frac{x\,dx}{(x^2-1)^{3/2}}$

$= -\frac{\ell n\,x}{\sqrt{x^2-1}} + \int \frac{1}{x}\frac{1}{\sqrt{x^2-1}}\,dx$

$= \sec^{-1} x - \frac{\ell n\,x}{\sqrt{x^2-1}} + c$

Sol.14 $I = \int \frac{2\sin 2\phi - \cos\phi}{6 - \cos^2\phi - 4\sin\phi}\,d\phi$

$= \int \frac{4\sin\phi\cos\phi - \cos\phi}{6-(1-\sin^2\phi) - 4\sin\phi}\,d\phi$

$= \int \frac{\cos\phi(4\sin\phi - 1)}{5 + \sin^2\phi - 4\sin\phi}\,d\phi$

put $\sin\phi = t \Rightarrow \cos\phi\,d\phi = dt$

$= \int \frac{(4t-1)}{t^2-4t+5}\,dt$

$= \int \frac{4t}{t^2-4t+5}\,dt - \int \frac{dt}{t^2-4t+5}$

$= 2\int \frac{2t-4+4}{t^2-4t+5}\,dt - \int \frac{dt}{t^2-4t+5}$

$= 2\int \frac{(2t-4)\,dt}{t^2-4t+5} + 7\int \frac{dt}{t^2-4t+5}$

$= 2\,\ell n\,(t^2-4t+5) + 7\int \frac{dt}{t^2-4t+5}$

$= 2\,\ell n\,(t^2-4t+5) + 7\int \frac{dt}{(t-2)^2+1}$

$= 2\,\ell n\,(t^2-4t+5) + 7\tan^{-1}(t-2) + c$

Sol.15 $I = \int \frac{dx}{1-\sin^4 x}$

$= \int \frac{dx}{(1-\sin^2 x)(1+\sin^2 x)} = \int \frac{\sec^2 x\,dx}{1+\sin^2 x}$

$= \int \frac{\sec^2 x.\sec^2 x.dx}{(\sec^2 x + \tan^2 x)} = \int \frac{\sec^2 x.\sec^2 x.dx}{(1+2\tan^2 x)}$

put $\tan x = t \Rightarrow \sec^2 x\,dx = dt$

$= \int \frac{(1+t^2)}{(2t^2+1)}\,dt = \frac{1}{2}\int \frac{(t^2+1/2+1/2)}{(t^2+1/2)}\,dt$

$= \frac{1}{2}\int dt + \frac{1}{4}\int \frac{dt}{t^2+1/2}$

$= \frac{1}{2}t + \frac{\sqrt{2}}{4}\tan^{-1}\sqrt{2}\,t + c$

$= \frac{1}{2}\tan x + \frac{1}{2\sqrt{2}}\tan^{-1}(\sqrt{2}\tan x) + c$

Sol.16 $I = \int \frac{\sqrt{4+x^2}}{x^6}\,dx$

$= \int \frac{x\sqrt{1+4x^{-2}}}{x^6}\,dx = \int \frac{1}{x^2}.\frac{\sqrt{1+4x^{-2}}}{x^3}\,dx$

$= \int \frac{x^{-2}\sqrt{1+4x^2}}{x^3}\,dx$

put $\quad 1 + 4x^{-2} = t^2 \Rightarrow \frac{dx}{x^3} = -\frac{t}{4}dt$

$= -\frac{1}{4}\int t^2\frac{(t^2-1)}{4}dt = -\frac{1}{16}\int (t^4 - t^2)\,dt$

$= -\frac{1}{16}\left[\frac{t^5}{5} - \frac{t^3}{3}\right] + c$

where $t = \sqrt{1+4x^{-2}}$

Sol.17 $I = \int \frac{1+x\cos x}{x(1-x^2e^2\sin x)}\,dx$

put $\quad 1 - x^2 e^{2\sin x} = t$

$[-2x\,e^{2\sin x} - x^2 e^{2\sin x}(2\cos x)]dx = dt$

$-2x\,e^{2\sin x}[1 + x\cos x]\,dx = dt$

$(1 + x\cos x)\,dx = \frac{dt}{2x\,e^{2\sin x}}$

$$I = -\int \frac{1}{t}\frac{dt}{2x^2\, e^{2\sin x}}$$

$$= -\frac{1}{2}\int \frac{1}{t}\frac{dt}{(1-t)} = \frac{1}{2}\int \frac{t+(1-t)}{t(1-t)}\,dt$$

$$= \frac{1}{2}\int \frac{dt}{1-t} + \frac{1}{2}\int \frac{dt}{t} = \frac{1}{2}\left[\ell n\, \frac{t}{1-t}\right] + c$$

Sol.18 $I = \int \underbrace{\cos 2x}_{II}\ \underbrace{\ell n\,(1+\tan x)}_{I}\,dx$

$$= \ell n\,(1+\tan x)\,\frac{\sin 2x}{2} - \int \frac{\sec^2 x}{1+\tan x}\cdot\frac{\sin 2x}{2}\,dx$$

$$= \frac{1}{2}\sin 2x\,\ell n\,(1+\tan x) - \int \frac{\sin x}{\sin x+\cos x}\,dx$$

$$= \frac{1}{2}\sin 2x\,\ell n\,(1+\tan x)$$

$$-\left[\frac{1}{2}\int \frac{\sin x+\cos x}{\sin x+\cos x}\,dx - \frac{1}{2}\int \frac{\sin x-\cos x}{\sin x+\cos x}\,dx\right]$$

$$= \frac{1}{2}\sin 2x\,\ell n\,(1+\tan x) - \frac{1}{2}x + \frac{1}{2}\ell n\,(\sin x+\cos x)+c$$

Sol.19 $I = \int \frac{1+\cos\alpha\cos x}{\cos\alpha+\cos x}\,dx$

$1 + \cos\alpha\cos x = A(\cos\alpha + \cos x) + B(-\sin x) + c$

$A = \cos\alpha,\ B = 0,\ C = \sin^2\alpha$

$$I = \int \cos\alpha\,dx + \sin^2\alpha\int \frac{dx}{\cos\alpha+\cos x}$$

$$= x\cos\alpha + \sin^2\alpha\int \frac{\sec^2\frac{x}{2}\,dx}{\cos\alpha\left(1+\tan^2\frac{x}{2}\right)+\left(1-\tan^2\frac{x}{2}\right)}$$

put $\tan\frac{x}{2} = t \Rightarrow \sec^2\frac{x}{2}\,dx = 2dt$

$$= x\cos\alpha + 2\sin^2\alpha\int \frac{dt}{\cos\alpha(1+t^2)+(1-t^2)}$$

$$= x\cos\alpha + 2\sin^2\alpha\int \frac{dt}{(\cos\alpha+1)-t^2(1-\cos\alpha)}$$

$$= x\cos\alpha + 2\sin^2\alpha\int \frac{dt}{2\cos^2\frac{\alpha}{2}-2\sin^2\frac{\alpha}{2}\,t^2}$$

$$= x\cos\alpha + \frac{\sin^2\alpha}{\sin^2\frac{\alpha}{2}}\int \frac{dt}{\cot^2\frac{\alpha}{2}-t^2}$$

$$= x\cos\alpha + \frac{\sin^2\alpha}{\sin^2\frac{\alpha}{2}}\,\frac{1}{2\cot\frac{\alpha}{2}}\,\ell n\left|\frac{\cot\frac{\alpha}{2}+t}{\cot\frac{\alpha}{2}-t}\right| + c$$

$$= x\cos\alpha + \sin\alpha\,\ell n\left|\frac{\cos\frac{1}{2}(\alpha-x)}{\cos\frac{1}{2}(\alpha+x)}\right| + c$$

Sol.20 $I = \int \cos x\,.\,e^x\,x^2\,dx$

$$I = \underbrace{\int \cos x\,e^x}_{II}\ \underbrace{x^2}_{I}\,dx$$

$$I = x^2\frac{e^x}{2}(\cos x+\sin x) - \int xe^x(\cos x+\sin x)\,dx$$

$$I = x^2\frac{e^x}{2}(\cos x+\sin x)$$

$$-\int x\,e^x\cos x\,dx - \int xe^x\sin x\,dx$$

$$I = x^2\frac{e^x}{2}(\cos x+\sin x)$$

$$-\frac{x\,e^x}{2}(\cos x+\sin x) + \frac{1}{2}\int e^x(\cos x+\sin x)\,dx$$

$$-\frac{x\,e^x}{2}(\sin x-\cos x) + \frac{1}{2}\int e^x(\sin x-\cos x)\,dx$$

$$I = x^2\frac{e^x}{2}(\cos x+\sin x) - xe^x\sin x + \int e^x\sin x\,dx$$

$$I = x^2\frac{e^x}{2}(\cos x+\sin x) - xe^x\sin x + \frac{e^x}{2}(\sin x-\cos x)+c$$

$$I = \frac{e^x}{2}\,[(x^2-1)\cos x + (x-1)^2\sin x] + c$$

Sol.21 $I = \int \frac{dx}{(x^3 + 3x^2 + 3x + 1)\sqrt{x^2 + 2x - 3}}$

$I = \int \frac{dx}{(x+1)^3\sqrt{(x+1)^2 - 4}}$

put $x + 1 = \frac{1}{t} \Rightarrow dx = -\frac{1}{t^2}dt$

$I = -\int \frac{t^2}{\sqrt{1-4t^2}}\,dt$

$I = \frac{1}{4}\int \frac{(1-4t^2)-1}{\sqrt{1-4t^2}}\,dt$

$I = \frac{1}{4}\int \sqrt{1-4t^2}\,dt - \frac{1}{4}\int \frac{dt}{\sqrt{1-4t^2}}$

$I = \frac{1}{4}\left[\frac{2t\sqrt{1-4t^2}}{2\times 2} + \frac{1}{2\times 2}\sin^{-1}(2t) - 1\sin^{-1}(2t)\right] + c$

$= \frac{1}{8}\sqrt{1-4t^2} - \frac{1}{16}\sin^{-1}2t + c$

$= \frac{1}{8}\frac{\sqrt{(x+1)^2-4}}{(x+1)} - \frac{1}{16}\left[\frac{\pi}{2} - \cos^{-1}\left(\frac{2}{x+1}\right)\right] + c$

$= \frac{\sqrt{x^2+2x-3}}{8(x+1)} + \frac{1}{16}\cos^{-1}\left(\frac{2}{x+1}\right) + c$

Sol.22 $I = \int e^x\left(\frac{x^3 - x + 2}{(x^2+1)^2}\right)dx$

$= \int e^x\left\{\frac{x^3+x^2+x+1}{(x^2+1)^2} + \left(\frac{-x^2-2x+1}{(x^2+1)^2}\right)\right\}dx$

$= \int e^x\left\{\underset{\substack{\uparrow\\ f(x)}}{\frac{x+1}{x^2+1}} + \underset{\substack{\uparrow\\ f'(x)}}{\left(\frac{-x^2-2x+1}{(x^2+1)^2}\right)}\right\}dx$

$= e^x\left(\frac{x+1}{x^2+1}\right) + c$

Sol.23 $I = \int \frac{\cos 2x - 3}{\cos^4 x\sqrt{4-\cot^2 x}}\,dx$

$= \int \frac{\cos^2 x - \sin^2 x - 3}{\cos^4 x\sqrt{4-\cot^2 x}}\,dx$

$= \int \frac{\sec^2 x(1-\tan^2 x - 3\sec^2 x)\,dx}{\sqrt{4-\cot^2 x}}$

$= \int \frac{\sec^2 x(1-\tan^2 x - 3(1+\tan^2 x))}{\sqrt{4-\cot^2 x}}\,dx$

$= \int \frac{\sec^2 x(-2-4\tan^2 x)\tan x}{\sqrt{4\tan^2 x - 1}}\,dx$

$4\tan^2 x - 1 = t^2 \Rightarrow \tan x\sec^2 x\,dx = \frac{t}{4}dt$

$I = \frac{1}{4}\int \frac{t(-2-(t^2+1))}{t}\,dt = \frac{1}{4}\int(-3-t^2)\,dt$

$I = \frac{1}{4}\left[-3t - \frac{t^3}{3}\right] + c \quad = \frac{1}{4}\left[-3-\frac{t^2}{3}\right]t + c$

$= \frac{1}{4}\left[-3 - \frac{(4\tan^2 x - 1)}{3}\right]\sqrt{4\tan^2 x - 1} + c$

$= \frac{1}{4}\left[\frac{-9-4\tan^2 x + 1}{3}\right]\sqrt{4\tan^2 x - 1} + c$

$$= \left[\frac{-2-\tan^2 x}{3}\right]\sqrt{4\tan^2 x - 1} + c$$

$$= -\frac{1}{3}(2+\tan^2 x)\sqrt{4\tan^2 x - 1} + c$$

Sol.24 $I = \int \sin 4x \; e^{\tan^2 x} dx$

$$= \int 2\sin 2x \cos 2x \; e^{\tan^2 x} dx$$

$$= \int \frac{4\tan x}{(1+\tan^2 x)} \frac{(1-\tan^2 x)}{(1+\tan^2 x)} . e^{\tan^2 x} dx$$

$$= \int \frac{4\tan x}{(1+\tan^2 x)^2} - \frac{4\tan x(\sec^2 x - 1)}{(1+\tan^2 x)^2} e^{\tan^2 x} dx$$

$$= \int \frac{8\tan x \sec^2 x}{(1+\tan^2 x)^3} - \frac{4\tan x \sec^2 x}{(1+\tan^2 x)^2} e^{\tan^2 x} dx$$

put $(1 + \tan^2 x) = t \Rightarrow 2\tan x \sec^2 x\, dx = dt$

$$= \frac{2}{e}\int \left(\underset{f'(x)}{\underset{\uparrow}{\frac{4}{t^3}}} - \underset{f(x)}{\underset{\uparrow}{\frac{2}{t^2}}}\right) e^t \, dt = \frac{2}{et^2} \; e^t + c$$

$$= \frac{2}{e(1+\tan^2 x)^2} \; e^{\tan^2 x} + c = \frac{2}{e}\cos^4 x \; e^{\tan^2 x} + c$$

Sol.25 $I = \int \underbrace{\tan^{-1} x}_{\mathrm{I}} . \underbrace{\ell n\,(1+x^2)\,dx}_{\mathrm{II}}$

$I = \tan^{-1} x \{x\, \ell n\,(1 + x^2) - 2x + 2\tan^{-1} x\}\, dx$

$$-\int \left\{\frac{x\,\ell n\,(1+x^2) - 2x + 2\tan^{-1} x}{1+x^2}\right\} dx$$

$I = x \tan^{-1} x \ln (1 + x^2) - 2x \tan^{-1} x + 2(\tan^{-1} x)^2$

$$-\int \frac{x\,\ell n(1+x^2)}{1+x^2} dx + \int \frac{2x}{1+x^2} dx - 2\int \frac{\tan^{-1} x}{(1+x^2)} dx$$

$I = x \tan^{-1} x\, \ell n\,(1 + x^2) - 2x \tan^{-1} x + 2(\tan^{-1} x)^2$

$-\frac{1}{4}\{\ell n\,(1 + x^2)\}^2 + \ell n\,(1 + x^2) - (\tan^{-1} x)^2$

$I = x \tan^{-1} x\, \ell n\,(1 + x^2) - 2x \tan^{-1} x + (\tan^{-1} x)^2$

$+ \ln (1 + x^2) - \left(\ell n\,(\sqrt{1+x^2})\right)^2 + c$

Sol.26 $I = \int e^x \frac{1+nx^{n-1} - x^{2n}}{(1-x^n)\sqrt{1-x^{2n}}} dx$

$$= \int e^x \left(\frac{(1-x^n)(1+x^n)}{(1-x^n)\sqrt{(1-x^n)(1+x^n)}} + \frac{nx^{n-1}}{(1-x^n)\sqrt{1-x^{2n}}}\right) dx$$

$$= \int e^x \left(\underset{f(x)}{\underset{\uparrow}{\sqrt{\frac{1+x^n}{1-x^n}}}} + \underset{f'(x)}{\underset{\uparrow}{\frac{nx^{n-1}}{(1-x^n)\sqrt{1-x^{2n}}}}}\right) dx$$

$$= e^x \sqrt{\frac{1+x^n}{1-x^n}} + c$$

Sol.27 $I = \int \frac{a + b\sin x}{(b + a\sin x)^2} dx$

$$= \frac{b}{a}\int \frac{\frac{a^2}{b} - b + (b + a\sin x)}{(b + a\sin x)^2} dx$$

$$= \frac{a^2 - b^2}{a} \int \frac{dx}{(b + a\sin x)^2} + \frac{b}{a}\int \frac{dx}{(b + a\sin x)}$$

Let $A = \frac{\cos x}{(b + a\sin x)} \Rightarrow \frac{dA}{dx} = \frac{-b\sin x - a}{(b + a\sin x)^2}$

$$\frac{dA}{dx} = -\frac{b}{a}\left\{\frac{a\sin x + b + \frac{a^2}{b} - b}{(b + a\sin x)^2}\right\}$$

$$\frac{dA}{dx} = -\frac{b}{a}\left\{\frac{1}{b + a\sin x} + \frac{a^2 - b^2}{b(b + a\sin x)^2}\right\}$$

$$A = \frac{-b}{a}\int\frac{dx}{b + a\sin x} - \frac{a^2 - b^2}{a}\int\frac{dx}{(b + a\sin x)^2}$$

$$\frac{a^2 - b^2}{a}\int\frac{dx}{(b + a\sin x)^2} = -\frac{b}{a}\int\frac{dx}{b + a\sin x} - A$$

$$I = \frac{b}{a}\int\frac{dx}{b + a\sin x} - A + \frac{b}{a}\int\frac{dx}{(b + a\sin x)}$$

$$I = -A + C$$

$$= -\left(\frac{\cos x}{b + a\sin x}\right) + c$$

Sol.28 $I = \int\frac{x\cos\alpha + 1}{(x^2 + 2x\cos\alpha + 1)^{3/2}}dx$

$$= \int\frac{x\cos\alpha + 1}{x^3\left(1 + \frac{2}{x}\cos\alpha + \frac{1}{x^2}\right)^{3/2}}dx$$

$$= \int\frac{\frac{\cos\alpha}{x^2} + \frac{1}{x^3}}{\left(1 + \frac{2}{x}\cos\alpha + \frac{1}{x^2}\right)^{3/2}}dx$$

$$1 + \frac{2}{x}\cos\alpha + \frac{1}{x^2} = t^2$$

$$\left(\frac{\cos\alpha}{x^2} + \frac{1}{x^3}\right)dx = -t\,dt$$

$$= -\int\frac{t\,dt}{t^3} = \frac{1}{t} + c = \frac{1}{\sqrt{\frac{2}{x}\cos\alpha + \frac{1}{x^2} + 1}} + c$$

$$= \frac{x}{\sqrt{x^2 + 2x\cos\alpha + 1}} + c$$

Sol.29 $I = \int\frac{\ell n\,(1 + \sin^2 x)}{\cos^2 x}dx$

$$= \int \sec^2 x\,\ell n\,(1 + \sin^2 x)dx$$

Using By parts

$$= \ell n\,(1 + \sin^2 x)\int\sec^2 x\,dx - \int\frac{2\sin x\cos x}{1 + \sin^2 x}.\tan x\,dx$$

$$= \tan x\,.\,\ell n\,(1 + \sin^2 x) - 2\int\frac{\sin^2 x}{1 + \sin^2 x}dx$$

$$= \tan.\ell n\,(1 + \sin^2 x) - 2x + 2\int\frac{dx}{1 + \sin^2 x}$$

$$= \tan.\ell n\,(1 + \sin^2 x) - 2x + 2\int\frac{\sec^2 x}{1 + 2\tan^2 x}dx$$

$\tan x = t \Rightarrow \sec^2 x\,dx = dt$

$$= \tan x.\ell n\,(1 + \sin^2 x) - 2x + 2\int\frac{dt}{1 + 2t^2}$$

$$= \tan x.\ell n\,(1 + \sin^2 x) - 2x + \int\frac{dt}{t^2 + \frac{1}{2}}$$

$$= \tan x.\ell n\,(1 + \sin^2 x) - 2x + \sqrt{2}\tan^{-1}(\sqrt{2}\,t) + c$$

$$= \tan x.\ell n\,(1 + \sin^2 x) - 2x + \sqrt{2}\tan^{-1}(\sqrt{2}\tan x) + c$$